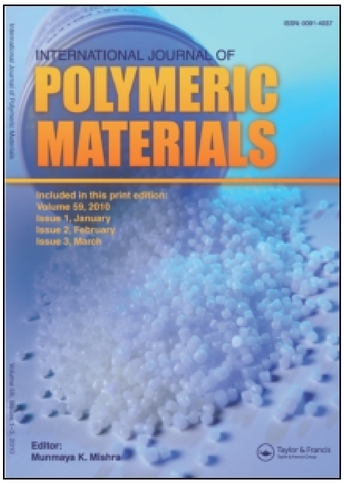

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Determination of Changes in Fracture Propagation from Fracture Curves

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A method for the calculation of a change in the local and translational velocity of crack propagation has been suggested from the shape and distribution of curves on the fracture surface. The relation between the calculated velocities and the fracture surface morphology has been proven for poly(methyl methacrylate).

INTRODUCTION

Fracture cracks in brittle polymeric materials usually originate and propagate in the craze[1,2] (Figure 1a). This is connected with the formation of secondary fractures which start propagating from places not yet attained by the main fracture front. The secondary fracture becomes the primary one for the following fracture; on meeting with the main fracture it becomes part of the latter.

Generally, the velocities of propagation of the main fracture front and of secondary fractures are different.[3,4] The formation of secondary fractures has as its consequence that in some places the secondary fracture crack propagates toward the main crack, so that on their meeting (merging of the secondary fracture crack with the main fracture crack) the main fracture crack "jumps" over a certain distance. Thus, the parameters of fracture curves (velocities of propagation of fractures, activation distances, distribution of centres of secondary fractures[4]) modify the course of the velocity of propagation of the main fracture crack.

It has been an aim of this paper to suggest a theoretical expression for the velocity of propagation of the main fracture crack from the parameters of

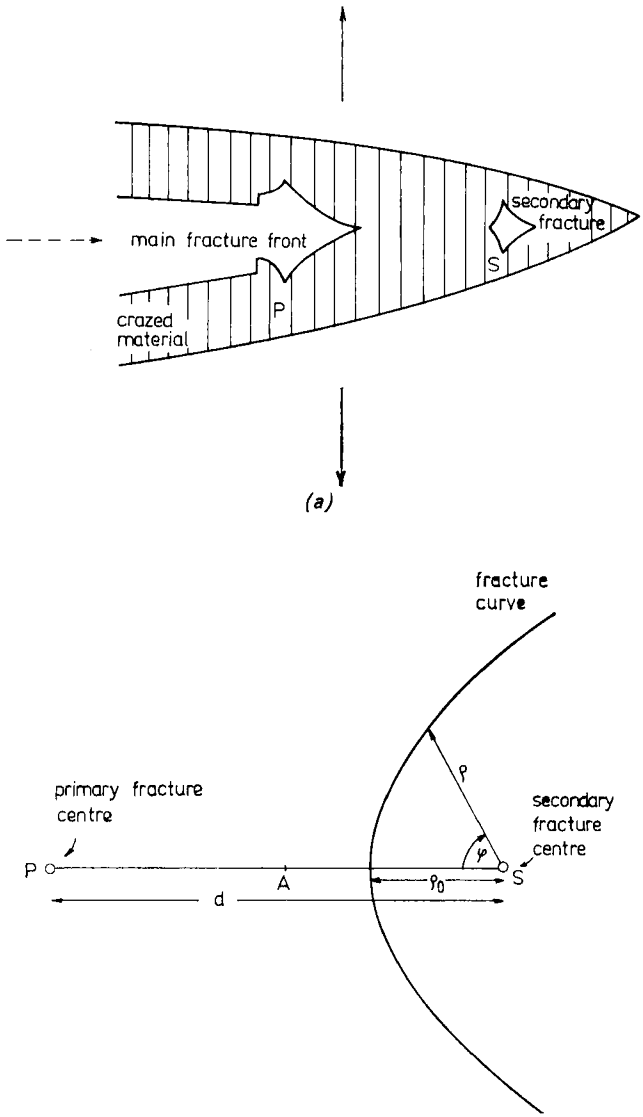

FIGURE 1 Schematic view of fracture crack propagating over secondary fractures (*a*) and definition of parameters of fracture curves (*b*) P and S are centres of the primary and or secondary crack, respectively. Full arrow indicates direction of the applied mechanical stress, broken arrow indicates direction of propagation of the fracture crack. Dimensions in direction of the applied stress are greatly magnified.

fracture curves and to apply the result to the calculation of changes in the velocity of the crack propagation from the morphology of the fracture surface of poly(methyl methacrylate).

CALCULATION OF THE VELOCITY OF PROPAGATION OF THE CRACK

The existence of secondary fracture processes is demonstrated by the occurrence of characteristic curves on the fracture surface[3,5–7] arising on the meeting of the primary crack with the secondary one (Figure 1b). The shape of the fracture curve depends on the velocities of propagation of both the primary and secondary fracture, on the distances between the centres of both fractures, d, and on the activation distance $a = \overline{AS}$, i.e. on the distance between the front of the primary fracture and the centre of the secondary fracture S at a moment when the secondary fracture starts spreading.[4]

If the assumptions of an isotropic propagation in planes perpendicular to the direction of the acting force and of the independence of the velocity of fracture cracks propagation of time are fulfilled for the fracture fronts, the equation

$$\rho^2(1-V^2)/d - 2\rho[V(1-a/d) + \cos\varphi] + a(2 - a/d) = 0 \qquad (1)$$

describing the shape of the fracture curve allows the determination (by using nonlinear regression method) of the basic parameters V, as needed for the evaluation of the course of the velocity of the fracture crack propagating over the secondary cracks.[4] V is the ratio of the respective velocities of propagation of the primary and secondary fractures, ρ and φ are polar coordinates of the fracture curve (Figure 1b). The distance between the fracture centres can be measured directly on the fracture surface. It is useful when calculating the velocity of the fracture crack propagation to employ the minimum value of the position vector ρ_0 (which can be measured directly and which together with the basic parameters V, a, d is bound by Eq. (1) expressed for $\varphi = 0$) instead of the activation distance a.

The validity of Eq. (1) for the shape of the fracture curves makes it possible to express the velocity of the fracture crack propagation. To calculate them one has to assume that the individual centres of secondary fractures lie on a straight line behind each other. Such an assumption is not too restricting, since in practice series of subsequent fracture curves are found quite frequently. Slight deviations from the distribution on the straight line are not important and the pathway of the fracture can be approximated by its projections onto the direction of propagation of the main fracture crack. The average velocity of fracture crack propagation from the beginning of the fracture (or from the starting point on the fracture surface) up to the place of origin of the ith secondary fracture differs from the average velocity of propagation of secondary fractures; the velocity increment corresponds to a shortening of the length of the fracture crack by intercepts in which the secondary fractures have propagated to meet the main fracture crack, and look as if they were "jumped over" when both fractures join.

A valuable information on the crack propagation is provided by the local fracture velocity, u_i, defined as the average rate at which the main fracture crack propagates over the region of the ith secondary fracture (i.e. it also includes the jump in the crack propagation).

If t_i is the time during which the fracture crack propagates from the centre of the primary fracture to a distance s_i (Figure 2), i.e. before the preceding

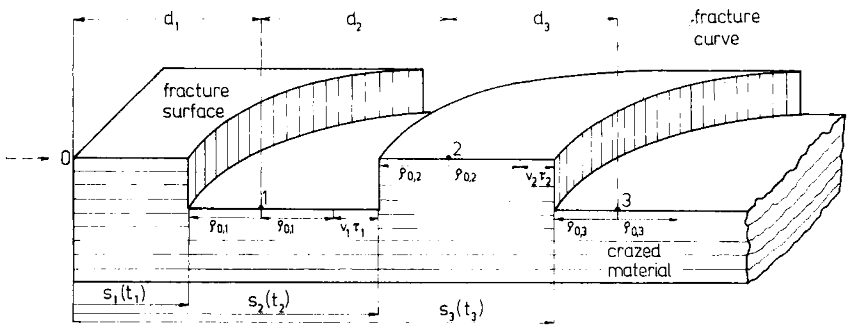

FIGURE 2 Schematic view of a series of fracture curves on fracture surface gradually arising on the straight line in the direction of propagation of the fracture crack. The direction of propagation is indicated by a broken arrow. Fracture is gradually formed from centres 0, 1, 2, 3; fracture curves have been made visible by cutting out a steplike relief in the meeting points of the individual fracture fronts.

crack joins the ith secondary fracture crack, and if τ_i is the time during which the propagating main fracture crack propagates over the region of the ith secondary fracture, it holds

$$t_i + \tau_i = t_{i+1} \tag{2}$$

with $i = 1, 2, 3, \ldots$. One can see from Figure 2 that

$$t_i = t_1 + \sum_{k=1}^{i-1} \tau_k \tag{3}$$

$$s_i = \sum_{k=1}^{i} d_k - \rho_{0,i} \tag{4}$$

$$\tau_k = \frac{d_{k+1} - (\rho_{0,k+1} + \rho_{0,k})}{v_k} \tag{5}$$

$k = 1, 2, 3, \ldots, \nu_k$ is the velocity of the fracture crack propagation of the kth secondary fracture. In accordance with Eq. (1) it holds

$$V_i = \nu_{i-1}/\nu_i, \tag{6}$$

and hence

$$\nu_i = \nu_0 / \prod_{k=1}^{i} V_k, \tag{7}$$

ν_0 being the initial propagation rate of the crack to which our measurements are related. According to the above definition,

$$u_i = (s_{i+1} - s_i)/\tau_i. \tag{8}$$

Eqs. (4) through 7 give the relative local velocity

$$u_i/\nu_0 = \frac{d_{i+1} - \rho_{0,i+1} + \rho_{0,i}}{(d_{i+1} - \rho_{0,i+1} - \rho_{0,i})\mathrm{L} \prod_{k=1}^{i} V_k}. \tag{9}$$

The variation of the ratio u_i/ν_0 in the individual places of the fracture surface indicates an acceleration or slowing-down of the propagation of the main fracture crack in the regions under investigation where secondary fracture cracks become operative.

In order to achieve correlation with macroscopic measurements of the velocity of crack propagation (determined from the time of crack propagation between two detection points) it is necessary to express the translational velocity of the crack (the average velocity of the main crack propagation). The translational velocity, w_i, is determined from the time t_i during which the crack propagates to a distance s_i:

$$w_i = s_i/t_i. \tag{10}$$

From Eqs (2) through (7) the relative translational velocity is expressed by

$$w_i/\nu_0 = \frac{\sum_{k=1}^{i} d_k - \rho_{0,i}}{d_1 - \rho_{0,1} + \sum_{k=1}^{i-1} \left\{ [d_{k+1} - (\rho_{0,k+1} + \rho_{0,k})] \prod_{l=1}^{k} V_l \right\}}. \tag{11}$$

Here, ν_0 again is the initial rate to which the measurements are related. For large i the relative translation velocity w_i/ν_0 approaches a certain limit affected by the average parameters of the fracture curves. The effect of the individual parameters on u_i/ν_0, w_i/ν_0 can be estimated from a model example of a hypothetical series of fracture curves with parameters chosen in advances (Figure 3), the same for all fracture curves ($\rho_{0,i} = \rho_0$, $V_i = V$, $d_i = 1$). The local velocity

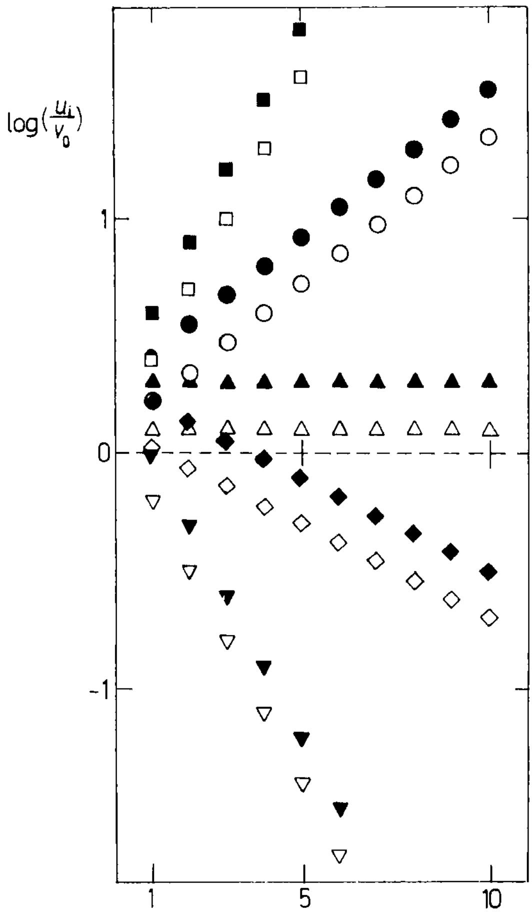

FIGURE 3a

u_i increases with increasing ρ_0 and decreases with increasing ratio of the velocities V. It can be seen from both Eq. (11) and Figure 3 that the translational velocity of crack propagation not only represents the individual elementary fracture processes, but depends on the distribution of the fracture centres and on the individul velocities of propagation of secondary fractures, and also on the place in which the transitional velocity is measured.

In a real case, however, the situation is complicated by the different character of all the three parameters, d_i, $\rho_{0,i}$, V_i of the individual fracture curves. To evaluate the course of the crack velocity (local or translational) it is necessary to measure d_i, $\rho_{0,i}$, and to determine V_i from the shape of the curve by non-linear regression analysis.[4]

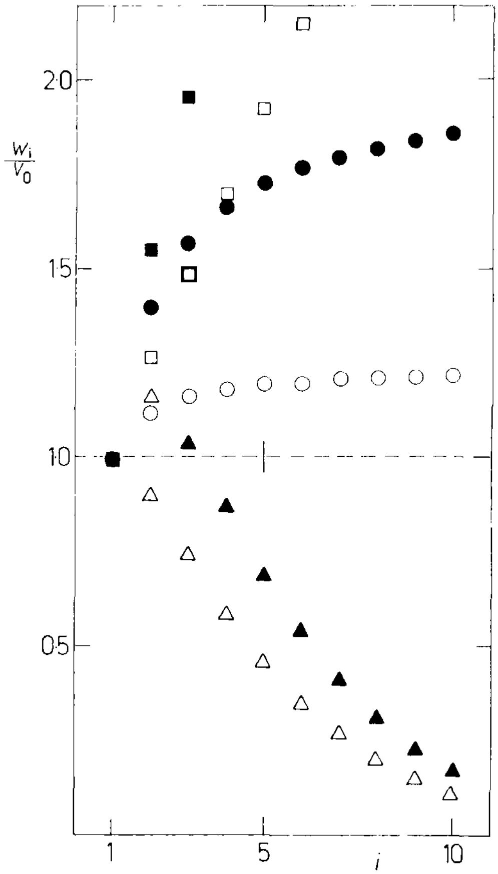

FIGURE 3b

FIGURE 3 Model example of the dependence of the relative velocity of the crack propagation on the number, *i*, of subsequent secondary fracture processes included in the calculations at constant parameters of fracture curves (*a*) logarithms of local relative velocity log (u_i/v_0) illustration of Eq. (9). Full symbols denote values corresponding to $\rho_0=0.25$; empty symbols $\rho_0=0.1$, $V=0.5$ ( ); 0.75 ( ); 1 (△); 1.2 ( ); 2 ( ); (*b*) translational relative velocity w_i/v_0 (illustration of Eq. (11)). Full symbols denote values corresponding to $\rho_0=0.25$; empty symbols $\rho_0=0.1$, $V=0.75$ (□); 1 (○); 1.5 (△). In all cases $w_i/v_0=1$ for $i=1$.

EXPERIMENTAL RESULTS

The relative local velocities and relative translational velocities of the fracture propagation were determined from a series of connected fracture curves due to fracture propagation in the impact test of poly(methyl methacrylate) (Figure 4). The fracture curves under investigation were chosen from the region of the

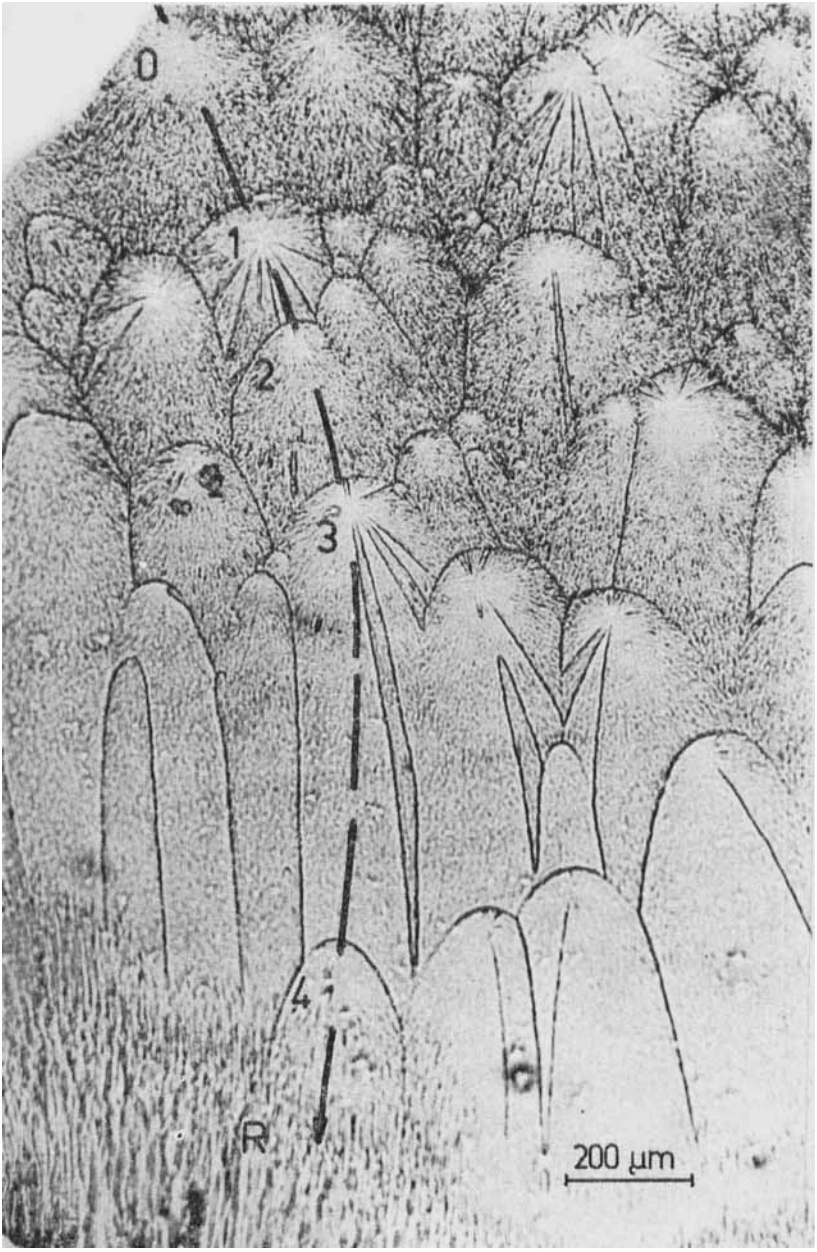

FIGURE 4 Light micrograph of a replica of fracture surface of poly(methyl methacrylate). Result of evaluation of the series of fracture curves is given in Table I.

fracture surface immediately preceding the formation of a rough area. It is obvious that both the shape and distribution of the fracture curves vary while approaching to the rough region R. This difference in the morphology of fracture curves is accompanied by very distinct changes in the parameters d_i, $\rho_{0,i}$, V_i, and also in the relative velocities of fracture propagation u_i/v_0,w_i/v_0 calculated from these parameters by using Eqs. (9) and (11). Table I).

TABLE I
Relative velocities of fracture propagation evaluated from a series of subsequent fracture curves in Figure 4

Curve (i)	d_i[a] μm	$\rho_{0,i}$[b] μm	V_i[c]	u_i/v_0[d]	w_i/v_0[e]
1	304	51	0·97	2·53	1·00
2	174	22	0·88	0·958	1·36
3	294	53	1·34	0·637	1·18
4	768	39	1·26	—	0·825

[a]Distance between the centres of the secondary fractures (distances were measured on the micrograph of the fracture surface);
[b]Distance between the centre of the secondary fracture and the top of the fracture curve;
[c]Ratio of the velocities of subsequent secondary fractures (cf. Eq. (6));
[d]Relative local velocity of fracture crack propagation;
[e]Relative translation velocity of fracture crack propagation.

DISCUSSION

The method of determination of the relative velocity (local and translational) of the fracture crack propagating over secondary cracks can be applied if the expression of the individual forming fracture curves by means of Eq. (1) is valid. Eq. (1) was derived assuming that fracture cracks propagate isotropically in planes not too distant from each other and perpendicular to the direction of the acting mechanical stress, and also that the propagation rates of the individual fracture cracks are constant. It appears, however, that the assumptions are fulfilled in most cases; their invalidity would be reflected in differences of the surface morphology inside fracture curves and in a deviation of the calculated shape of the curve from the real one.[4]

The acceleration in the propagation of the fracture front depends on the magnitude of the part of the fracture path which is "jumped over", that is, the magnitude of $\rho_{0,i}/d_i$, and also on the other fracture parameters (cf. Eqs. (9) and (11). The effect is best illustrated by a model example of subsequent fractures (Figure 3): both the relative local velocity and the relative transitional velocity of crack propagation increase with increasing $\rho_{0,i}/d_i$. A lasting acceleration of the propagating crack with respect to the velocity of v_0 the reference crack occurs owing to "jumps" already at $V = 1$; a temporary acceleration (in the proximity of the beginning of the fracture crack pathway) appears even in that case when the secondary fracture propagates slower than the primary one, i.e. $V > 1$, if $\rho_0 > d(V - 1)/2V$. In a real case when the parameters of the fracture curves differ for subsequent fractures the oscillating values of d_i, $\rho_{0,i}$, V_i give no clear indication of the trend, which on the other hand is distinctly seen from the calculated relative velocities (Table I).

Pronounced changes in the relative local velocity of propagation of the fracture crack can be expected in those places of the fracture surface where its morphology changes.

A calculation carried out by the procedure suggested above showed that already at a distance of 2 mm from the relatively coarser region *R* (Figure 4) the fracture crack propagation is slowed down. This phenomenon could be explained by the energy consumation before the region *R* by the initiation of a large number of secondary fractures in the region *R*.

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